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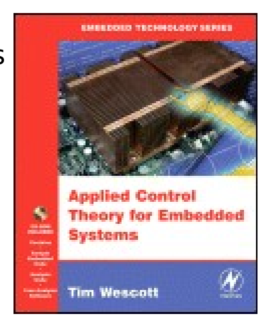
Errata

as of December 18th, 2012

Applied Control Theory for Embedded Systems is written for the practicing engineer who needs to develop working control systems without going back to school for three years. It is aimed directly at software engineers who are learning control theory for the first time, however it also covers many real-world issues in much greater depth than is taught in a 'theory' course.



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1 http://www.powells.com/biblio/0750678399?&PID=30696

Found a Problem?

Thanks to all of the people who have already written to me with reports of typographical or just plain logical errors in this book. Your alertness and willingness to help out have improved the quality of this book for all.

If you find a problem in the book and you don't see it addressed in this errata, or if some explanation is unclear, please send me mail:

Email: tim@wescottdesign.com

Errata

Chapter 1

On page 4, in the second paragraph under "Sensor", at the second sentence: "... voltage to be read by a DAC" should be: "... voltage to be read by a ADC".

Page 9 should end with:

1.6 Support & Upgrades

> For up-to-date information on this book, including errata and fresh content, see http://www.wescottdesign.com/actfes/actfes.html.

Chapter 2

Equation 2.3 is a bit confusing, because I was mixing a common notation for the time index, *k*, with a common notation for integrator gain, k_i. This equation will be more clear if it reads

$$y(n)=k_i \sum_{p=0}^{n} x(p) \tag{2.3}$$

Equation 2.8 does not match the text, and should read:

$$x_k = a_1 x_{k-1} + a_2 x_{k-2} + b_1 u_{k-1} + b_2 u_{k-2} \tag{2.8}$$

Equation 2.9 should read:

$$x_k = \sum_{k=1}^{K} a_{K-k} x_{n-k} + \sum_{k=0}^{K} b_{K-k} u_{n-k} \tag{2.9}$$

In Example 2.1, the text after equation 2.13 should read:

> In typical differential-equation fashion, the nonhomogeneous solution of (2,11) echos the form of the input. Since the input is constant for all n > 0, so is the nonhomogeneous solution.

Example 2.1 is incomplete. The derivation of a_0 is presented, but the derivation of a_1 and a_2 is not. This is how it is done:

> To find a_1 and a_2 we need only solve the system of equations

$$\begin{aligned} x_0 &= a_0 + a_1 + a_2 \\ x_1 &= a_0 + 0.9\,a_1 + 0.95\,a_2 \end{aligned} \quad . \tag{2.15a}$$

To do this we need to know x_0 and x_1. Since we know that $x_{-2} = 0$ and $x_{-1} = 0$ we can easily find these values by hand:

$$\begin{aligned} x_0 &= 1.85\,x_{-1} - 0.855\,x_{-2} + 0.005\,u_0 \\ x_1 &= 1.85\,x_0 - 0.855\,x_{-1} + 0.005\,u_1 \end{aligned} \quad . \tag{2.15b}$$

Substituting the proper values in to these expressions gives

$$\begin{aligned} x_0 &= 1.85(0) - 0.855(0) + 0.005(1) = 0.005 \\ x_1 &= 1.85(0.005) - 0.855(0) + 0.005(1) = 0.01425 \end{aligned} \quad . \tag{2.15c}$$

These can be substituted back into (2.15a) to give

$$\begin{aligned} 0.005 &= 1 + a_1 + a_2 \\ 0.01425 &= 1 + 0.9\,a_1 + 0.95\,a_2 \end{aligned} \tag{2.15d}$$

Solving for a_1 and a_2 gives

$$\begin{aligned} a_1 &= 0.81 \\ a_2 &= -1.805 \end{aligned} \tag{2.15e}$$

for an overall solution of

$$x_n = \begin{cases} 0 & n < 0 \\ 1 + 0.81(0.9)^n - 1.805(0.95)^n & n \geq 0 \end{cases} \tag{2.15f}$$

Equation (2.18) is a classic geometric series, but I didn't point this out.

Equation (2.27) should read (with the z in the denominator):

$$A_1 = \lim_{z \to a_1} \left[\frac{z - a_1}{z} X(z) \right] \tag{2.27}$$

Equation (2.35) lost its z in the denominator, also; the corrected version reads:

$$\lim_{z\to a_2}\left[\frac{d}{dz}\frac{(z-a_2)^2}{z}X(z)\right]=\lim_{z\to a_2}\frac{d}{dz}\left[\frac{(z-a_2)^2 A_0}{z}+\frac{(z-a_2)^2 A_1}{z-a_1}+\frac{(z-a_2)^2 A_2}{(z-a_2)^2}+\frac{(z-a_2)^2 A_3}{z-a_2}\right]$$

$$\lim_{z\to a_2}\left[\frac{d}{dz}\frac{(z-a_2)^2}{z}X(z)\right]=\lim_{z\to a_2}\frac{d}{dz}\left[\frac{(z-a_2)^2 A_0}{z}+\frac{(z-a_2)^2 A_1}{z-a_1}+A_2+(z-a_2)A_3\right] \qquad (2.35)$$

$$\lim_{z\to a_2}\left[\frac{d}{dz}\frac{(z-a_2)^2}{z}X(z)\right]=A_3$$

Equation (2.36) should read as below (i.e. you should evaluate the derivative at $z = a$). This equation gives the residual for the m^{th} repeated root per equation (2.35).

$$A_m=\lim_{z\to a}\left(\frac{1}{(n-m)!}\frac{d^{n-m}}{dz^{n-m}}\frac{(z-a)^n}{z}X(z)\right) \qquad (2.36)$$

Equation (2.40) in table 2.1 should read as below.

$k^2 u(k)$	$\frac{z^2+z}{(z-1)^3}$	$k^2 u(k)$ is a parabola – note the less than simple numerator	(2.40)

The motivated reader could have verified this using equation (2.41), however it should read

$k^n u(k)$	$\lim_{b\to 0}\left[(-1)^n \frac{d^n}{db^n}\frac{z}{z-e^b}\right]$	This is nasty: I'm still looking for a better one.	(2.41)

Equation (2.43) is also in error:

$k\, d^k u(k)$	$\frac{d\, z}{(z-d)^2}$	*d* must be a constant	(2.43)

The notation in Example 2.4 is inconsistent: the signal *x* in (2.77) is the shaft output angle *θ* in Figure 2.1, and the signal *u* in (2.77) is the motor input voltage.

Equation (2.79) in Example 2.4 should read

$$X(z)\left[1-(1+a)z^{-1}+a z^{-2}\right]=U(z)\,b\,z^{-1} \qquad (2.79)$$

In going from Equation (2.82) to (2.83) I left out the factor of four not once, but twice. This equation should read

$$x_k=\left(k-4+4(0.8)^k\right)u(k) \qquad (2.79)$$

Equation (2.85) in Example 2.5 should read as below. Note that 'forced' in this context is the same thing as a nonhomogeneous solution to a differential equation.

$$y(k)=y_F(k)+A(0.8)^k+B(0.9+j\,0.17)^k+B^*(0.9-j\,0.17)^k \qquad (2.85)$$

Note that in Equation (2.86), *C* is real; we have eliminated the complex numbers by going to the expression that uses a phase-shifted sine wave.

Equation (2.88), the Euler identity for sin *x*, should read

$$x(k)=\sin(\theta k)=\frac{j}{2}\left(e^{-j\theta k}-e^{j\theta k}\right) \qquad (2.88)$$

This correction can be carried through to (2.89):

$$y_h(k)=A\sin(\theta k+\phi)=A\,\frac{j}{2}\left(e^{-j(\theta k+\phi)}-e^{j(\theta k+\phi)}\right) \qquad (2.89)$$

Chapter 3

Some readers have questioned where Equation 3.1 comes from. It is the transfer function of a 1st-order low pass system, which is a common model for simple systems. The meaning of the term 'low-pass' will be come clear in section 3.2, and is described in table 3.1 on page 56.

Equation (3.3) is simply equation (3.2) written for the specific time when the output of the system is equal to 0.1 (which, because this system's unit step response has a final value of 1 is 10% of the final value). As written, Equation (3.3) is confusing: the pole value is being raised by the time delay in ticks (t_d). Adding appropriate parentheses clarifies this:

$$1-d^{(t_d)}=0.1 \qquad (3.3)$$

Equation (3.4) is derived from Equation (3.3) by first solving for t_d, then finding the closest integer fit for t_d. Unfortunately, as written it is just plain wrong, because yours truly was confused. t_d is the time delay in ticks, but may be some fractional amount that misrepresents the amount if time it really takes, so it needs to be rounded up with the ceiling function. (3.4) should read:

$$k_d=ceil(t_d)=ceil\left(\frac{\ln(0.9)}{\ln(d)}\right) \qquad (3.4)$$

where *ceil* is the C-library ceiling function that finds the largest integer that is not lower than the argument.

Equation (3.9) dropped one possible term; it should read

$$z=e^{-a-j\theta},e^{-a+j\theta} \qquad (3.9)$$

or, alternatively it could read

$$z=e^{-a\pm j\theta} \qquad (3.9)$$

Equation (3.11) dropped an absolute value sign; it should read:

$$y_k=2|A|d^k\cos(\theta k+\phi) \qquad (3.11)$$

Footnote 2 on page 47 should contain the arctangent instead of tangent; it should read “For example, $\phi = \tan^{-1}\left(\frac{\mathrm{Im}(A)}{\mathrm{Re}(A)}\right)$ “

The captions for figures 3.3 and 3.4 should read

Figure 3.3 Rise and settling times as a function of the damping ratio

Figure 3.4 Overshoot as a function of the damping ratio

Equation (3.20) on page 59 (in Example 3.1) has the wrong scaling. It should read

$$H_{lp}(z) = \frac{0.001 z^2}{(z-0.9)^3} \tag{3.20}$$

Chapter 4

Table 4.1, pp 70-71: A number of blocks are missing their bottom borders. These blocks should all resemble the block in Figure 4.4.

Figure 4.11, p 75: The feedback signal, *Tp*, from the output on the right to the summation junction with T_T on the left is a very pale gray. It should match the rest of the signal lines.

Figure 4.16, p82: The G2 and G1 + G2 blocks are missing their bottom borders.

I missed a term in the denominator of the second half of equation 4.44 on page 87. It should read:

$$\begin{gathered} T_d(z) = \frac{Y(z)}{U_d(z)} = \frac{k_{pp}(z-1)}{(z-d_1)(z-d_2)(z-1) + k_h(k_p z k_i - k_p) k_{pp}}, \\ T_d(z) = \frac{k_{pp}(z-1)}{z^3 - (1+d_1+d_2) z^2 + (d_1 d_2 + d_1 + d_2 + k_h k_{pp} k_p) z + k_h k_{pp}(k_i - k_p) - d_1 d_2} \end{gathered} \tag{4.44}$$

Equation (4.51) on page 90 not only has a missing exponent but the numbers in the denominator were evaluated incorrectly. This should read:

$$\frac{U_c}{U} = \frac{9.5\mathrm{z}^3 - 20.27\mathrm{z}^2 + 13.80\mathrm{z} - 3.035}{z^3 - 1.9665\mathrm{z}^2 + 1.2628\mathrm{z} - 0.2601} \tag{4.51}$$

This propagates to Equation (4.52):

$$U_{cstep} = \frac{z}{z-1} \frac{9.5\mathrm{z}^3 - 20.27\mathrm{z}^2 + 13.80\mathrm{z} - 3.035}{z^3 - 1.9665\mathrm{z}^2 + 1.2628\mathrm{z} - 0.2601} \tag{4.52}$$

It must have been a transcription error, because equation 4.52 is correct. Equation 4.54, however, has the error:

$$U_{cramp} = \frac{z}{(z-1)^2} \frac{9.5z^3 - 20.27z^2 + 13.80z - 3.035}{z^3 - 1.9665z^2 + 1.2628z - 0.2601} \quad (4.54)$$

Equation 4.55 reads incorrectly, but evaluating it with the correct denominator results in the answer given:

$$U_{final} = \lim_{z \to 1} \frac{z}{z-1} \frac{9.5z^3 - 20.27z^2 + 13.80z - 3.035}{z^3 - 1.9665z^2 + 1.2628z - 0.2601} = 49.9 \quad (4.55)$$

Chapter 5

Figure 5.1, p 97 – the caption should read *Figure 5.1 Root locus plot of (5.1)*

The text right before Figure 5.5 on page 103 mentions a '+' marking the point on the root locus in Figure 5.5 where the system has a double pole. This mark doesn't show on the figure; the point in question is where the circular part of the root locus hits the real number line at approximately z = 0.65.

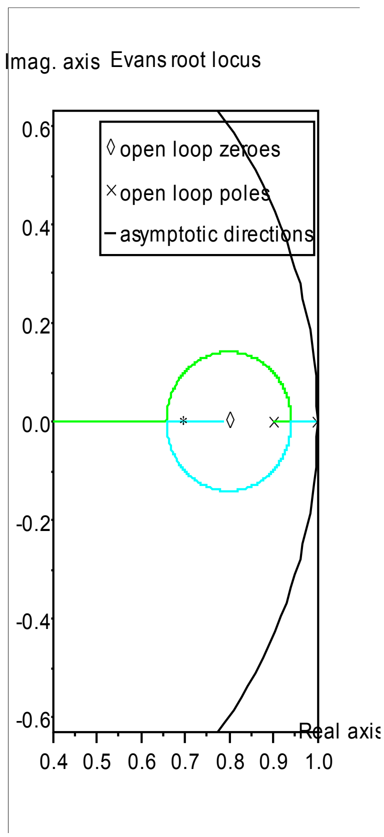

Figure 5.5 The Updated Root Locus

Equation (5.15) on page 105 can be misleading for some readers: the numerator in (5.15) is the same as the numerator in (5.14). This is mere happenstance. As an alternative example we might choose to vary k_3. In this case (5.14) would become

$$G(z)=\frac{(k_1 k_2)z}{\left[z^3-2z^2+(1+k_1 k_2)z\right]+k_1 k_3(z^2-2z+1)} \quad . \tag{5.14a}$$

When the denominator is split up the fictitious open loop gain becomes

$$G_{fol}(z)=\frac{k_1 k_3(z^2-2z+1)}{z^3-2z^2+(1+k_1 k_2)z} \tag{5.15a}$$

Figure 5.9 in Example 5.3 on page 110 is missing a line: the signal going into the block with gain '2' is connected to the output of the left-most summation.

The paragraph between figures 5.10 and 5.11 should have a phrase that reads "closed-loop response around 5Hz", not "1/2 Hz"

Figure 5.13 on page 114 should have a circle of radius ½ inscribed around the point *z* = -1; were the Nyquist plot to touch this circle it would indicate that the sensitivity was reaching 6dB; were the Nyquist plot to go inside of this circle it would indicate that the sensitivity was exceeding 6dB. The correct figure is shown below.

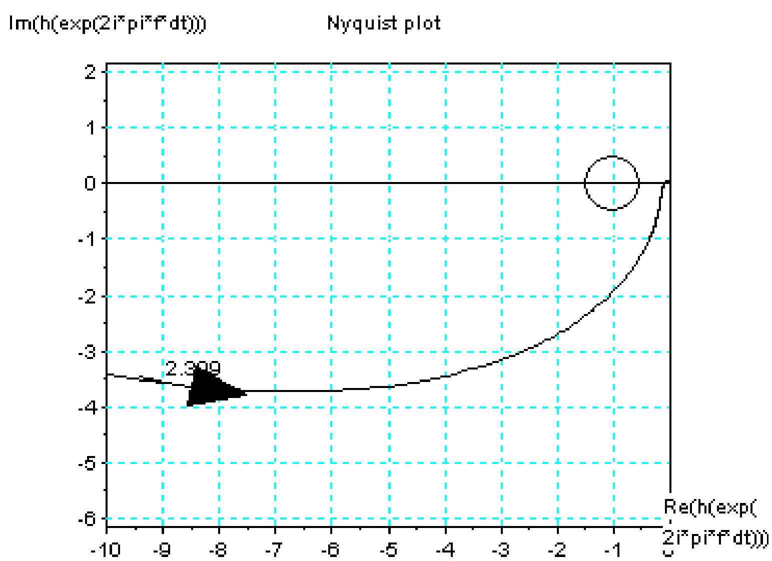

Figure 5.13: A Nyquist Plot of the Response Plotted in Figure 5.10.

Figure 5.14 may be confusing for some readers: the 'x' marks indicate the roots of an arbitrary polynomial, *not* the poles of a transfer function.

In numerous places in Chapter 5 '1' is mentioned where the correct value is '-1'. This appears to be a pervasive error in the way my editor treats hyphens. Here are the spots that I know about:

- The second paragraph on page 109 should read:

  When a system has an open loop gain of -1 its gain is 0dB and its phase shift is 180°. Thus, at a frequency where the system's open loop phase shift is exactly 180° the system is only a gain change away from having a gain of exactly -1; the amount of gain change (in dB) required to get you to 0dB of gain at such a frequency is the gain margin. Similarly, at a frequency where the system's open loop gain magnitude is exactly one the system is only a phase shift away from having a gain of exactly -1; the amount of phase change required to get you to 180° at such a frequency is the phase margin.

- The final paragraph on page 113 should read:

Figure 5.13 shows a Nyquist plot of the open-loop system response shown in the Bode plot in Figure 5.10. As with any other frequency design method the important consideration for both robustness and stability is how closely the system's complex gain approaches -1.

- The end of the first paragraph (after equation 5.30) and and the second paragraph on page 122 (after the heading "Gain and Phase Margins) should read:

... where G_{ol} is the open-loop gain. This means that the sensitivity of the system at any point is just the reciprocal of the distance between -1 and G_{ol} at the frequency in question. Thus, you can easily see the maximum sensitivity of the system by looking for the closest approach of the trace to -1. In Figure 13 a circle of radius ½ has been inscribed on the plot, indicating a maximum sensitivity of two times (6dB) the "natural" plant sensitivity. Such a circle, at whatever diameter you deem appropriate, is a useful aid to graphical system design where you can adjust the controller parameters while keeping an eye on whether the open-loop gain is violating some predefined sensitivity limit.

Gain and Phase Margins

Determining system gain and phase margins is fairly simple with the Nyquist plot. Recall that the gain margin is the amount of gain change needed to bring the loop gain to -1 when it is a pure negative number, and that the phase margin is the amount of phase change needed to bring the loop gain to -1 when the absolute value of the loop gain is 1.

At the end of the one paragraph on page 123, the phase should say "-110 degrees", not 110 degrees.

Chapter 6

The block to the right in the forward path of the diagram in Figure 6.7 is missing its fraction bar; this transfer function should be

$$\frac{0.01}{z-1}$$

None of the axes on the bode plots are labeled. In all cases the bottom axis is the frequency, either in Hz or in fractions of the sampling rate. The vertical axis on the left is the gain in dB, and corresponds to the bold black line; the vertical axis on the right is the phase shift in degrees, and corresponds to the light black or dashed black line.

Chapter 8

Equation (8.40) should read

$$x_n = \begin{cases} y_{min} - k_p u_n & x_{n-1} + (k_i + k_p) u_n < y_{min} \\ y_{max} - k_p u_n & x_{n-1} + (k_i + k_p) u_n > y_{max} \\ x_{n-1} + k_i u_n & \text{otherwise} \end{cases} \tag{8.40}$$

$$y_n = x_n + k_p u_n + \text{differential term}$$

Chapter 9

Sharp-eyed readers will note that there is a discrepancy between equation 9.7 and the discussion in the paragraph that follows. Equation 9.7 is correct, however the discussion was written for the equation

$$A(f)\, e^{j\Phi(f)} = \frac{2}{A_T N} \sum_{k=0}^{N-1} y(k)\, e^{\left(2\pi f k - \frac{\pi}{2}\right)} \quad , \tag{9.7a}$$

which is equal to (9.7) – the difference is notational, where the *j* term in the denominator of (9.7) is taken into the exponent of *e* in the equation above.

Chapter 10

Equation 10.14, p 267: The equation should appear as k sub d equals quantity z minus 1 over z.

$$H(z) = k_d \frac{z-1}{z} \tag{10.14}$$

Listing 10.6, p 267: The caption should read :

Listing 10.6 Two ways of implementing differentiators with integers

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